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Where two fractals meet: The scaling of a self-avoiding walk on a percolation cluster

C. von Ferber,[1,2,*] V. Blavats'ka,[3,†] R. Folk,[2,‡] and Yu. Holovatch[2,4,§]
[1]*Theoretische Polymerphysik, Universität Freiburg, D-79104 Freiburg, Germany*
[2]*Institut für Theoretische Physik, Johannes Kepler Universität Linz, A-4040, Linz, Austria*
[3]*Institute for Condensed Matter Physics, National Academy of Sciences of Ukraine, UA-79011 Lviv, Ukraine*
[4]*Institute for Condensed Matter Physics and Ivan Franko National University of Lviv, UA–79011 Lviv, Ukraine*

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The scaling properties of self-avoiding walks on a d-dimensional diluted lattice at the percolation threshold are analyzed by a field-theoretical renormalization group approach. To this end we reconsider the model of Y. Meir and A. B. Harris [Phys. Rev. Lett. **63**, 2819 (1989)] and argue that via renormalization its multifractal properties are directly accessible. While the former first order perturbation did not agree with the results of other methods our analytic result gives an accurate description of the available MC and exact enumeration data in a wide range of dimensions $2 \leqslant d \leqslant 6$.

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Polymers and percolation clusters are among the most frequently encountered examples of fractals in condensed matter physics [1–3]. They have been studied extensively since the mid 70's but important questions remain open, in particular the long-standing discrepancy between the field-theoretical and numerical results for the scaling of a self-avoiding walk (SAW) on a percolation cluster. The latter is at the heart of the geometrical interpretation of thermal critical phenomena [4]. In our special case it is furthermore connected with the appearance of multifractality in field theories. The purpose of this paper is twofold: to resolve the discrepancy between the field-theoretical and numerical results and to contribute a field-theoretical framework for the multifractality recently discussed for this problem [13,20].

When a long polymer chain is immersed in a good solvent its mean-square end-to-end distance $\overline{R^2}$ scales with the monomer number N as:

$$\overline{R^2} \sim N^{2\nu_{\mathrm{SAW}}}, \qquad N \to \infty \tag{1}$$

with the exponent $\nu_{\mathrm{SAW}}(d)$ which depends on the (Euclidean) space dimension d only. This scaling of polymers (1) is perfectly described by the SAW on a *regular* d-dimensional lattice [1] and the fractal dimension of a polymer chain readily follows: $d_{\mathrm{SAW}}=1/\nu_{\mathrm{SAW}}$. For space dimensions d above the upper critical dimension $d_{\mathrm{up}}=4$ the scaling exponent becomes trivial: $\nu_{\mathrm{SAW}}(d>4)=1/2$, whereas for $d<d_{\mathrm{up}}$ the non-trivial dependence on d is described, e.g., by the phenomenological Flory formula [1] $\nu_{\mathrm{SAW}}=3/(d+2)$. This found its further support by the renormalization group (RG) $\tilde{\varepsilon}=4-d$-expansion known currently to the high orders [5]: $\nu_{\mathrm{SAW}}=1/2+\tilde{\varepsilon}/16+15\tilde{\varepsilon}^2/512+\cdots$.

When a SAW resides on a *disordered* (quenched diluted) lattice–such a situation might be experimentally realized studying a polymer solution in a porous medium, but is of its own interest as well–the asymptotic scaling behavior is a more subtle matter [6–8]. Numerous MC simulations [9–14] and exact enumeration studies [15–21], which date back to the early 1980s [8], lead to the conclusion that there are the following regimes for the scaling of a SAW on a disordered lattice: (i) weak disorder, when the concentration p of bonds allowed for the random walker is higher than the percolation concentration p_{PC} and (ii) strong disorder, directly at $p=p_{\mathrm{PC}}$. By further diluting the lattice to $p<p_{\mathrm{PC}}$ no macroscopically connected cluster, "percolation cluster," remains and the lattice becomes disconnected. In regime (i) the scaling law (1) is valid with the same exponent ν_{SAW} for the diluted lattice independent of p, whereas in case (ii) the scaling law (1) holds with a new exponent $\nu_{\mathrm{p}} \neq \nu_{\mathrm{SAW}}$. A hint to the physical understanding of these phenomena is given by the fact that weak disorder does not change the dimension of a lattice visited by a random walker, whereas the percolation cluster itself is a fractal with fractal dimension dependent on d: $d_{\mathrm{PC}}(d)=d-\beta_{\mathrm{PC}}/\nu_{\mathrm{PC}}$, where β_{PC} and ν_{PC} are familiar percolation exponents [2]. In this way, $\nu_{\mathrm{SAW}}(d)$ must change along with the dimension d_{PC} of the (fractal) lattice on which the walk resides. A modified Flory formula [9] for the exponent of a SAW on the percolation cluster $\nu_{\mathrm{p}}=3/(d_{\mathrm{PC}}+2)$ along with results of similar theoretical studies [22–29] gives numbers in astonishing agreement with the data observed (see Table I). Since $d_{\mathrm{up}}=6$ for percolation [2], the exponent $\nu_{\mathrm{p}}(d \geqslant 6)=1/2$ [33].

Although the Flory-like theories [22–29] offer good approximations for $\nu_{\mathrm{p}}(d)$ in a wide range of d, even more astonishing is the fact that up to now there do not exist any satisfactory theoretical estimates for $\nu_{\mathrm{p}}(d)$ based on a theory, which takes into account non-Markovian properties of the SAW, a task which was completed for regular lattices in the mid-1970s [1]. Existing real-space RG studies [15,22,30,34] give satisfactory estimates for $d=2$, whereas the field-theoretical approaches aimed to describe the situation at higher dimensions lead to contradictory conclusions. In particular, the field theory developed in Ref. [15] supported $d_{\mathrm{up}}=6$ and presented a calculation of ν_{p} in the first order of

*Email address: ferber@physik.uni-freiburg.de
†Email address: viktoria@icmp.lviv.ua
‡Email address: folk@tphys.uni-linz.ac.at
§Email address: hol@icmp.lviv.ua

TABLE I. The exponent ν_{p} for a SAW on a percolation cluster. FL: Flory-like theories, EE: exact enumerations, RS, RG: real-space and field-theoretic RG. The first line shows ν_{SAW} for SAW on the regular lattice ($d=2$ [31], $d=3$ [32]).

d	2	3	4	5	6
ν_{SAW}	3/4	0.5882(11)	1/2	1/2	1/2
FL, [22]	0.778	0.662	0.593	0.543	1/2
[23]	0.69(1)	0.57(2)	0.49(3)		1/2
[24]		0.70(3)	0.63	0.56	1/2
[25]	0.770	0.656	0.57	0.52	1/2
[26]	0.76	0.65	0.58		1/2
[27]	0.75–0.76	0.64–0.66	0.57–0.59	0.55–0.57	1/2
[28]	0.77	0.66	0.62	0.56	1/2
MC, [10]	$\simeq\nu_{\mathrm{SAW}}$	0.612(10)			
[11]	$\simeq\nu_{\mathrm{SAW}}$	0.605(10)			
[12]	0.77(1)				
[13]	0.783(3)				
[14]		0.62–0.63	0.56–0.57		
EE, [15]	0.76(8)	0.67(4)	0.63(2)	0.54(2)	
[16]	0.81(3)				
[16]	0.745(10)	0.635(10)			
[17]		0.65(1)			
[18]	0.745(20)	0.640(15)			
[19]	0.770(5)	0.660(5)			
[20]	0.778(15)	0.66(1)			
[20]	0.787(10)	0.662(6)			
RS, [30]	0.767				
[22]	0.778	0.724			
RG, [15]	0.595	0.571	0.548	0.524	1/2
(10)	0.785	0.678	0.595	0.536	1/2

$\varepsilon=6-d$. However the numerical estimates obtained from this result are in poor agreement with numbers observed by other means, leading in particular to the surprising estimate $\nu_{\mathrm{p}} \simeq \nu_{\mathrm{SAW}}$ in $d=3$ (see Table I). In turn, Ref. [35] even questioned the renormalizability of this field theory and suggested another theory with $d_{\mathrm{up}}=4$ which is obviously disproved by computer simulations and exact enumerations at dimensions $d=4,5$ [14,15].

There is another important reason why the scaling of a SAW on a percolation cluster calls for further theoretical study. As it became clear now, higher-order correlations of a fractal object at another fractal lead to multifractality [36]. Recently studied examples of multifractal phenomena are found in such different fields as diffusion in the vicinity of an absorbing polymer [37], random resistor networks [38], quantum gravity [39]. A SAW on a percolation cluster is a good candidate to possess multifractal behavior. While indications for such a behavior are found in computer simulations [20], it is implicit and quantitatively described by our RG scheme.

Let us consider a diluted lattice with sites $\mathbf{x}_i$ in terms of variables $p_{ij}=0,1$ that indicate whether a given bond between the sites $\mathbf{x}_i$ and $\mathbf{x}_j$ is present or not. To describe the

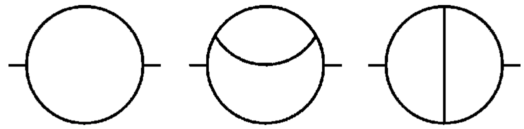

FIG. 1. The Feynman graphs of the vertex function $\Gamma^{(2)}(q)$ in the two lowest orders.

critical properties of SAWs on this lattice following the idea of de Gennes [1] we introduce m-component spin variables $S_\alpha(\mathbf{x}_i)$, $\alpha=1,\ldots,m$, and evaluate the theory for $m=0$. To allow for the averaging over the *quenched* disorder the spins are n-fold replicated which gives for the Hamiltonian:

$$e^{-\mathcal{H}_S} = \left\langle \exp\left\{ -\frac{K}{2}\sum_{\langle i,j\rangle} p_{ij} \sum_{\alpha=1}^{m}\sum_{\beta=1}^{n} S_\alpha^\beta(\mathbf{x}_i) S_\alpha^\beta(\mathbf{x}_j)\right\}\right\rangle_p \tag{2}$$

where we sum over the nearest neighbors i, j and denote by $\langle\cdots\rangle_p$ the average over the random variables p_{ij} which take the value 1 and 0 with probabilities p and $(1-p)$, respectively, and K is an interaction parameter. In the following we will work with a field theoretical representation of the effective Hamiltonian defined in (2). This is achieved [15] via a Stratonovich-Hubbard transformation to tensor fields $\psi_k(\mathbf{x})$ with components $\psi_{k;\beta_1,\ldots,\beta_k}^{\alpha_1,\ldots,\alpha_k}(\mathbf{x})$ conjugated to the product $\Pi_{j=1}^{k} S_{\alpha_j}^{\beta_j}(\mathbf{x})$ of k components of the replicated spin with $\beta_1<\cdots<\beta_k$. This results in the effective Hamiltonian up to order ψ^3 [15]:

$$\mathcal{H}_\psi = \frac{1}{2}\int d^d q \sum_k (r_k+q^2)\psi_k(\mathbf{q}):\psi_k(-\mathbf{q}) + \frac{w}{6}\int d^d x\, \psi^3(\mathbf{x}), \tag{3}$$

where $\psi_k(\mathbf{q})$ is the Fourier transform of $\psi_k(\mathbf{x})$, w is the coupling constant, and the inner product reads:

$$\psi_k(\mathbf{q}):\psi_k(-\mathbf{q}) = \sum_{\{\alpha_i\}}\sum_{\{\beta_i\}} |\psi_{k;\beta_1,\ldots,\beta_k}^{\alpha_1,\ldots,\alpha_k}(\mathbf{q})|^2,$$

and $\psi^3(\mathbf{x})$ is a symbolic notation for a product of three ψ_k fields. Only those cubic terms ψ^3 are allowed for which all pairs (α_i,β_i) appear exactly twice. A second condition on the diagrammatic contributions to perturbation theory can be derived from the de Gennes limit $m=0$, namely, if any index (α,β) appears only on the internal propagator of a diagram, then its contribution vanishes.

We note the unusual dependence of "masses" r_k on k. This is reminiscent of the fact that in the $m=0$ limit the theory (2) has a multitude of critical points in the $n=0$ replica limit contrary to the $m>0$ cases [15,40]. This has impact on the renormalization of the theory (3) as we will show in the following.

We choose to calculate the critical properties of the theory by analyzing its vertex functions, in particular $\Gamma^{(2)}(q)$, $\Gamma^{(3)}(\{q\})$, and $\Gamma^{(2,1)}(\{q\})$ where the latter includes an insertion of the $\psi:\psi$ operator. Each of these Γ-functions will depend on the family of masses $\{r_k\}$. The Feynman graphs of the contributions to the two-point vertex function $\Gamma^{(2)}(q)$ in the two lowest orders are shown in Fig. 1. The contributions to

$\Gamma^{(2,1)}$ are found from this by placing an insertion on each of the inner propagator lines. These integrals are evaluated then in dimensional regularization in dimension $d=6-\varepsilon$ and minimal subtraction [41] using a Laurent-expansion in ε. Usually the renormalization of the vertex functions is defined in terms of Z-factors in such a way that the products $Z_\psi\Gamma^{(2)}$, $Z_w\Gamma^{(3)}$, $Z_{\psi^2}\Gamma^{(2,1)}$ are free of ε-poles. However, the insertion of the $\psi:\psi$-operator together with the k-dependence of the masses r_k leads to the following renormalization procedure. The vertex function $\Gamma^{(2,1)}$ even when evaluated at zero mass remains k-dependent:

$$\Gamma^{(2,1)} = \Gamma_0^{(2,1)} + k\Gamma_1^{(2,1)} + k^2\Gamma_2^{(2,1)} + \cdots \quad (4)$$

and it cannot be renormalized by one multiplicative Z-factor. The essential feature of this expansion is that each term shows a different scaling behavior. In this way the multitude of critical points recognized already by Derrida [40] and Meir and Harris [15] manifests itself in our present formalism and leads to a *spectrum of exponents*. Instead of a single Z-factor Z_{ψ^2} a whole family of factors $Z^{(i)}_{\psi:\psi}$ is necessary to renormalize all $\Gamma_i^{(2,1)}$ in (4). This allows one to define the β- and η -functions

$$\beta(w) = \kappa\frac{d}{d\kappa}\ln Z_w, \quad \eta(w) = \kappa\frac{d}{d\kappa}\ln Z_\psi, \quad (5)$$

that describe the RG-flows with respect to the rescaling parameter κ and are the same as for the ψ^3 Potts model [42]. Furthermore,

$$\eta^{(i)}_{\psi:\psi}(w) = \kappa\frac{d}{d\kappa}\ln Z^{(i)}_{\psi:\psi} \quad (6)$$

govern the anomalous scaling of the corresponding functions $\Gamma_i^{(2,1)}$.

The explicit calculations proceed as follows: (i) One starts with the vertex function $\Gamma^{(2)}_{\psi_k}$ corresponding to the propagator of the field ψ_k. (ii) For the masses one inserts the expansion $r_k=\mu\Sigma_{j=0}^{\infty} u_j k^j$. (iii) The insertion of $\psi:\psi$ is defined by the derivative $(\partial/\partial\mu)\Gamma^{(2)}_{\psi_k}$ evaluated at zero mass for $\mu=0$. (iv) Performing the summation over the replica indices the contributions to the different $\Gamma_i^{(2,1)}$ are generated by rearranging the expansion in k. The requirement of multiplicative renormalization for each $\Gamma_i^{(2,1)}$ fixes the coefficients u_j in the expansion for r_k.

Following this procedure we obtain $\varepsilon=6-d$ expansions for the spectrum of exponents $\eta^{(i)}_{\psi:\psi}$ at the stable fixed point w^* with $\beta(w^*)=0$:

$$\eta^{(0)}_{\psi:\psi} = -2\varepsilon/7 - 167\varepsilon^2/24, \quad (7)$$

$$\eta^{(1)}_{\psi:\psi} = -\varepsilon/7 - 604\varepsilon^2/9261, \quad (8)$$

$$\eta^{(2)}_{\psi:\psi} = -3\varepsilon/14 - 113443\varepsilon^2/1926288. \quad (9)$$

Substituting (7) and (8) into $1/\nu^{(i)}=2-\eta+\eta^{(i)}_{\psi:\psi}$ together with the known result [42] $\eta=-\varepsilon/21-206\varepsilon^2/21^3$ we recover $\nu^{(0)}\equiv\nu_{\rm PC}$ [42] and extend the first-order expression [15] for ν_p by

$$\nu^{(1)} \equiv \nu_{\rm p} = 1/2 + \varepsilon/42 + 110\varepsilon^2/21^3. \quad (10)$$

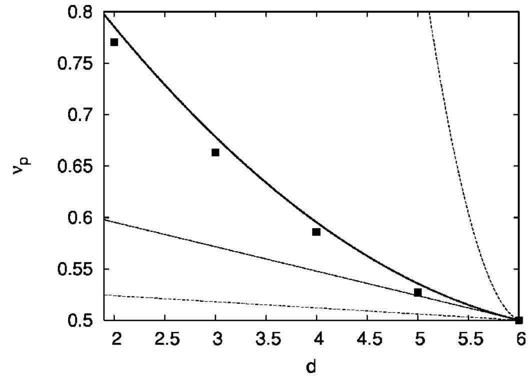

FIG. 2. The correlation exponent $\nu_{\rm p}$. Bold line: (10), thin line: one-loop result [15], closed boxes: Flory result $\nu_{\rm p}=3/(d_{\rm PC}+2)$ with $d_{\rm PC}$ from [44]. Exponents for the shortest and longest SAW on percolation cluster [45] are shown by dotted lines.

Equation (10) presents our first main result, the implication of which we discuss below. The second outcome of our analysis is the higher spectrum of exponents (6) which quantitatively describes the multifractality of the problem and concerns the lth moments $\langle\overline{N^l}(\mathbf{x}_1,\mathbf{x}_2)\rangle_p$ of the mean number $\overline{N}(\mathbf{x}_1,\mathbf{x}_2)$ of steps of SAWs between the sites $\mathbf{x}_1$ and $\mathbf{x}_2$ [15] which scale as $\langle\overline{N^l}(\mathbf{x}_1,\mathbf{x}_2)\rangle_p \sim |\mathbf{x}_1-\mathbf{x}_2|^{l/\nu^{(l)}}$ [43]. Note, however, that the higher moments of the end-to-end distance $\overline{R^{2l}}$, for which no multifractality was found in Ref. [20], have nothing to do with the scaling of $\overline{N^l}$ [15]. Evaluating the result for $\nu_{\rm p}$ (10) by direct substitution of $\varepsilon=6-d$ one finds nearly perfect correspondence with available MC and exact enumeration results over the range $d=2,\ldots,5$, see Table I. This presents a *qualitative* improvement over the linear result as seen in Fig. 2 where we also show that the result is in between the limits given by the shortest and longest SAWs on percolation cluster [45].

A rather peculiar finding is that results of the phenomenological Flory-like formulas evaluated using the fractal characteristics of the percolation cluster are numerically very close to our result in the same region of dimensions. The ambiguity [19] in defining a Flory-like scheme however leads to the different results in Table I.

Since the ψ^3 theory as applied to the present problem has the upper critical dimension $d_{\rm up}=6$ non-trivial scaling follows for dimensions $d=4,5$, which is out of reach of the approach of Ref. [35] relying on a ϕ^4-theory with $d_{\rm up}=4$ and thus trivial scaling at and above $d=4$.

From the physical point of view, our result for the exponent $\nu_{\rm p}$ together with the data of exact enumeration (EE) and Flory-like theories (see Table I) predicts a swelling of a polymer coil on the percolation cluster with respect to the pure lattice: $\nu_{\rm p}>\nu_{\rm SAW}$ for $d=2-5$. Up to now, this phenomenon has clearly been observed only in MC simulations for $d=2$

[13]. Although simulations on $d=3$ percolation clusters have been claimed to show this effect [9–11,14], these studies were subsequently criticized for using inappropriate data analysis [10,16,21] and for lack of accuracy. At $d=3$ our formula (10) predicts a 13% increase of ν_{p} with respect to ν_{SAW} which is larger than at $d=2$ (5%) and should be more easily observed by current state-of-art simulations. Given that even at $d=2$ we are in nice agreement with MC and EE data and the reliability of the perturbative RG results increases with d, this number calls for verification in MC experiments of similar accuracy.

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